

10 Straight lines and hyperbolas

FORECAST AND FUNCTIONS

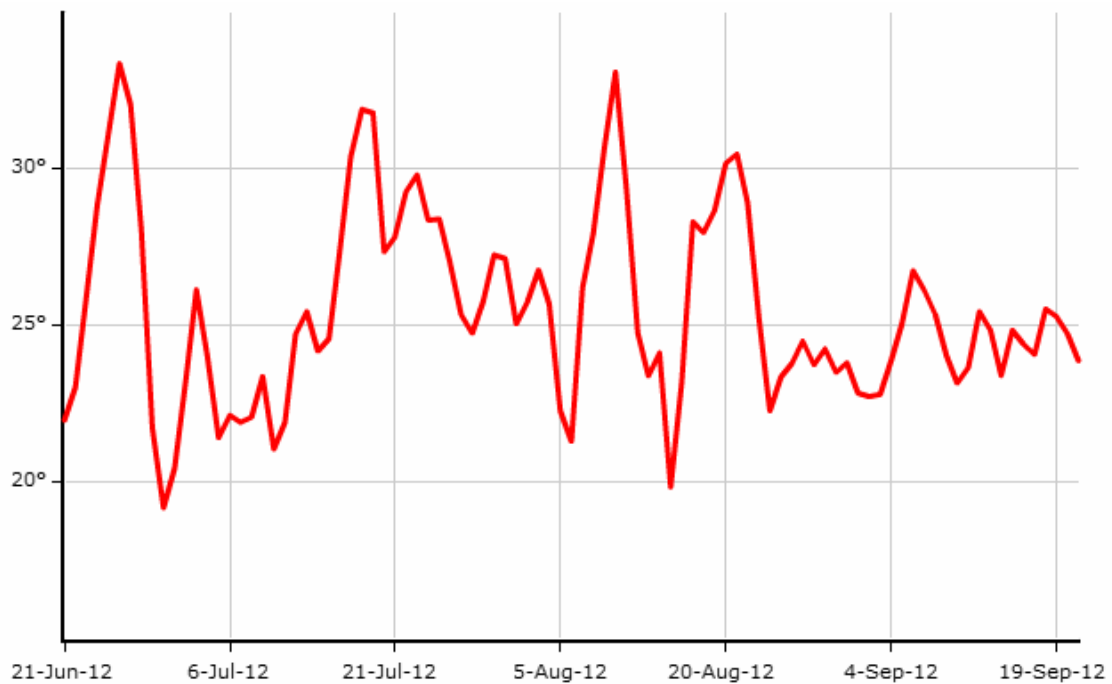
The data are from the website www.eltiempo.es

The forecast uses a lot of graphs in order to inform about the weather.

The path for the page is: website -> Cáceres province -> Cáceres city -> history

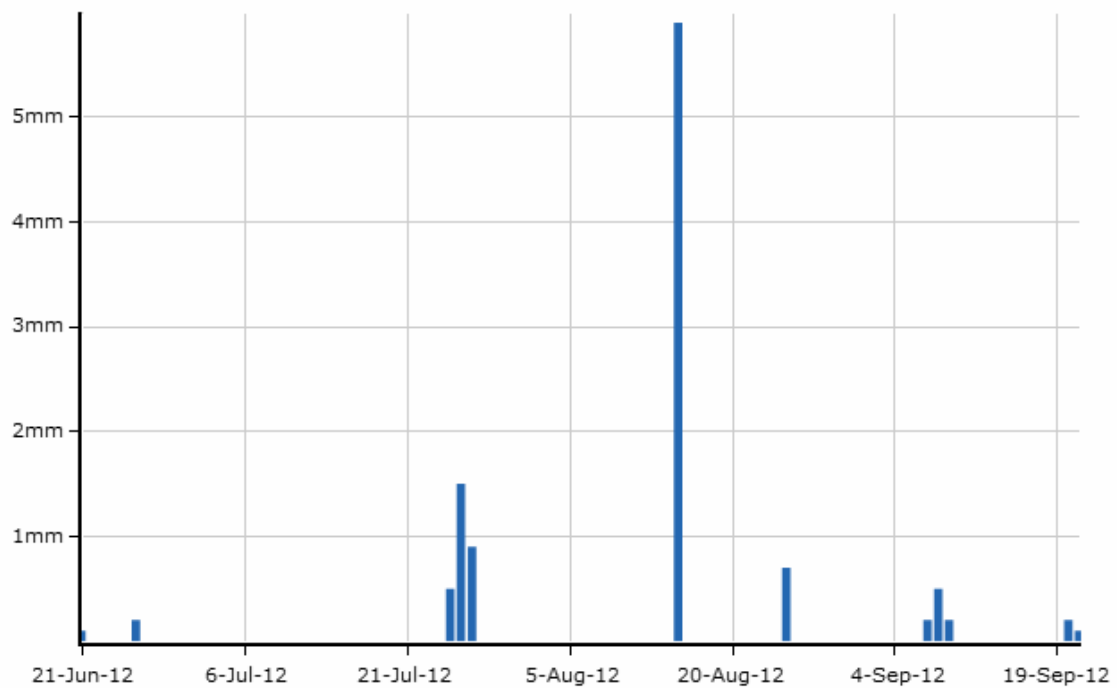
We will look at some of them about Cáceres weather in the summer of 2012 and get some conclusions.

TEMPERATURES



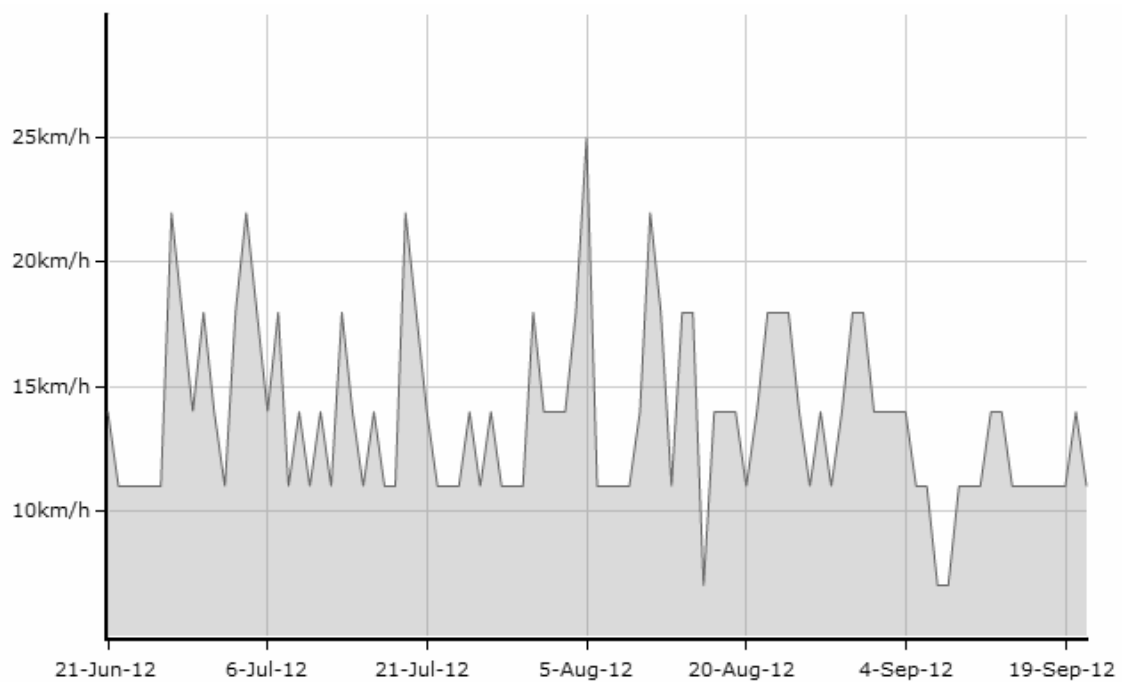
This graph shows the average temperatures during the summer of 2012 in Cáceres city.

PRECIPITATIONS



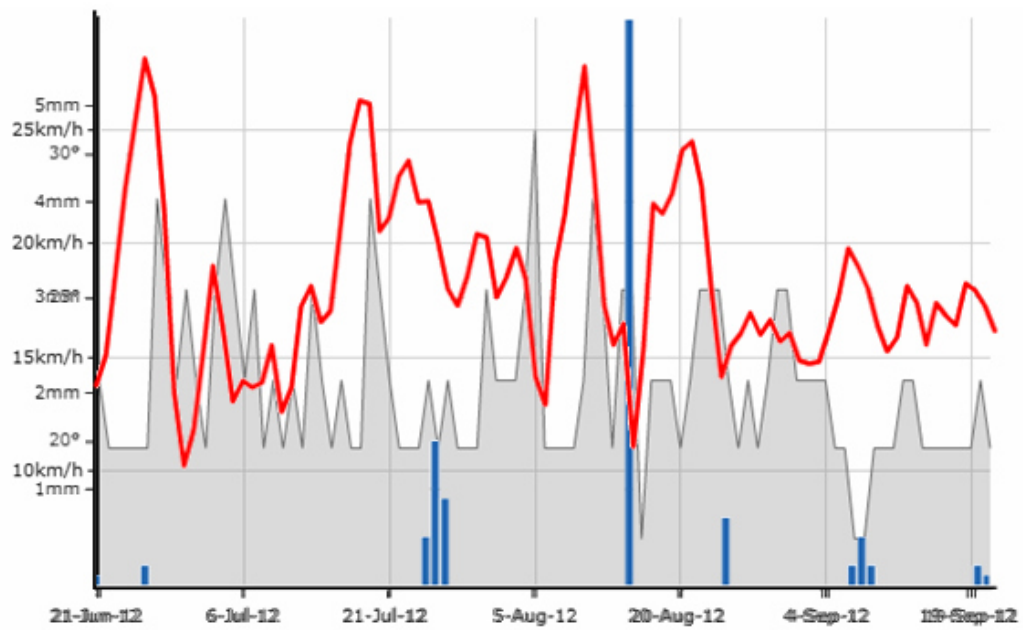
This graph shows the number of mm / m² a day.

WIND



This graph shows the wind speed.

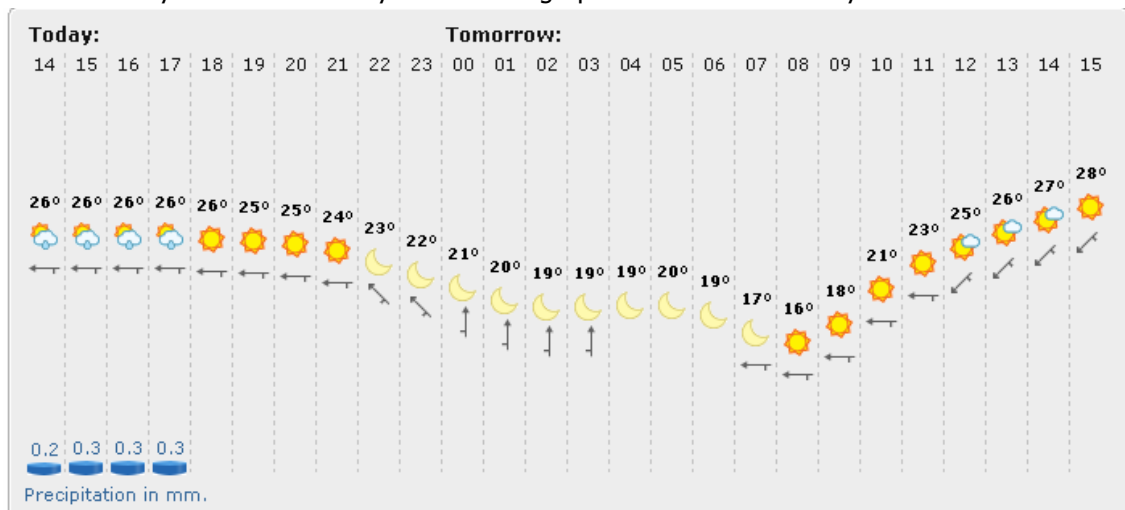
TEMPERATURE, RAIN AND WIND TOGETHER



Do you get some conclusion?

PER HOUR

Finally we will see a very informative graph about a summer day in Cáceres.



Could you explain the different features we can observe? A very interesting thing is the smooth changes in every variable.

The sun and the moon

The arrows

The temperatures

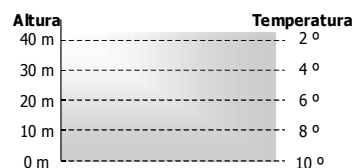
The blue cylinders and the numbers above of them.

The hour

1. INFORMATION PROCESSING

Science is based on the study of information we collect from a specific event in order to discover relationships.

To represent the information we can use: tables and graphs.



TABLES

A table is a method to write the related values in an experiment or event. We use two rows or two columns with the related pair values.

Make a table from the information of the picture.

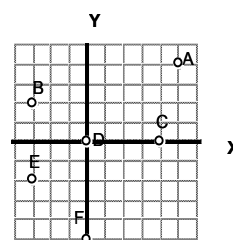
CARTESIAN COORDINATES

To represent information with two variables we use two numerical perpendicular straight lines with a common origin. They are the coordinate axes.

The horizontal axis is called abscissa axis and we write X. We represent the first variable on it.

The vertical axis is called ordinate axis and we write Y. We represent the second variable on it.

Each pair of values is represented by a point whose coordinates are (x, y).



2. FUNCTION

Functions are the mathematical tools we use to express situations of change, variation and movement.

For example, the length of a spring and the hanging weight; the position of the Earth in the sky and the day are functions.

Variable

It is a numerical value that changes. The opposite is a constant. Since they are variables we write them with letters. Temperature, pressure, time, number of mobile phones are variables.

Function

It is the numeric relation between two linked variables in an event.

The variable we use as reference is the independent variable. We use 'x' to name it.

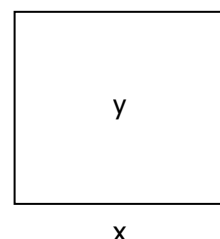
The variable we are studying is the dependent variable. We use 'y' to name it.

For example, $y = x^2$ is the relation between the side $-x-$ and the surface area $-y-$ in a square.

The usual form to write a function is $y=f(x)$, which means 'y' is function of 'x'.

For example, the price of a taxi trip is a function of the distance travelled.

The price of an orange bag is a function of the weight.



	Independent (x)	--- Function >>>	Dependent (y)
Square	Side	$y = x^2$	Area
Taxi	Distance	$y = 3x + 2$	Price
Oranges	Weight	$y = 1'5 \cdot x$	Price

TABLE, GRAPH AND FORMULA

There are three ways to describe a function: a table, a graph and a formula.

Table

A table is a method to describe a function. We use two rows or two columns with the related pair values.

Complete the chart for the function $y = x^2$ –area of a square according to its side–

x	y
0	
1	
2	
3	

Graph

It is a visual way to show the relation states by a function by using coordinate axes.

Coordinate axes

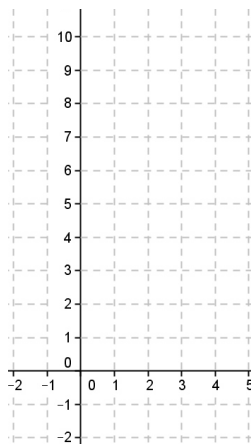
To represent information with two variables we use two perpendicular lines with a common origin. They are the coordinate axes.

The horizontal is called abscissa axis and we write X. We represent the first variable on it.

The vertical is called ordinate axis and we write Y. We represent the second variable on it.

Each pair of values is represented by a point whose coordinates are (x, y).

Represent the above function on the grid. That is the relation between the side of a square and its area. You have to use the above chart.



Formula

It is the most useful and precise way to express a function. It shows the numeric relation between the two variables.

$y = x^2$ is the formula.

Discrete and continuous variables

A variable is discrete if it takes isolated values.

For example, the number of sons in a family is a discrete variable because it only can take isolated values: 0, 1, 2, 3... The number of mobile phones is a discrete variable too.

A variable is continuous if it takes all the values in an interval.

For example, the height of a person is continuous because it can take all the possible values between two of them. The temperature is a continuous variable too.

3. LINEAR FUNCTION. DIRECT PROPORTIONALITY

It is the relationship between two magnitudes which are in direct proportion.

For example, we buy oranges €0.5 kilo. What is the constant of proportionality? Write some related values, for instance, the prices we pay for 1 kg; 2 kg; 3 kg...

$$x = 1 \text{ kg}; y = 0.5 \cdot 1 = \text{€}0.5$$

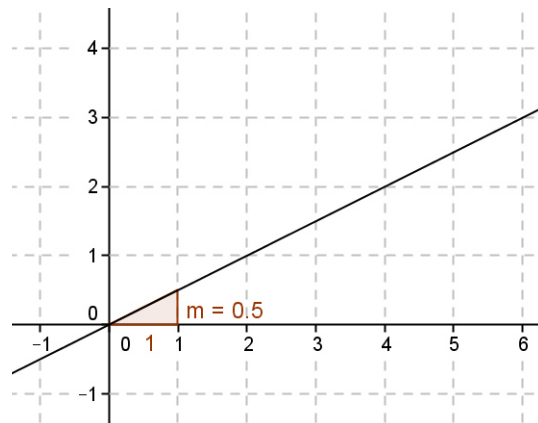
$$x = 2 \text{ kg};$$

$$x = 3 \text{ kg};$$

GRAPH

The graph for a linear function is a straight line through the origin of coordinates.

This is the graph for $y = 0.5x$



The constant of proportionality is the slope of the straight line. This is a measure of its inclination. Here it is $m = 0.5$

The slope is the ratio between whatever related values 'y' and 'x' because it is a proportion; and we represent it by 'm': $m = \frac{y}{x}$

The slope is 0.5 in the example.

EQUATION OR FORMULA

The equation or formula is simple: $y = mx$.

'm' is the slope or gradient. It is the constant of proportionality.

$m > 0$ means the function is increasing.

$m < 0$ means a decreasing function.

Write the formula that relates the weight and the price in the example.

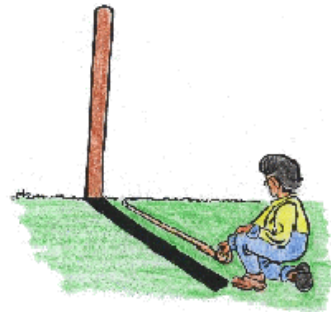
FUNCTION FROM ONE POINT

One point or one data is enough information to get the formula for a direct proportion. If I know how much a weight of oranges is, I will know the formula immediately.

Let's say $A(x, y)$ is whatever point or data from the function (chart). We get the constant of proportionality in this

$$\text{way: } k = \frac{y}{x}.$$

For example, you measure at 12 o'clock the shadow of a post 4 m long –x–. If the shadow length is 8 m –y– could you deduce the formula that relates the length and the shadow at this hour?



4. AFFINE FUNCTION

DEFINITION

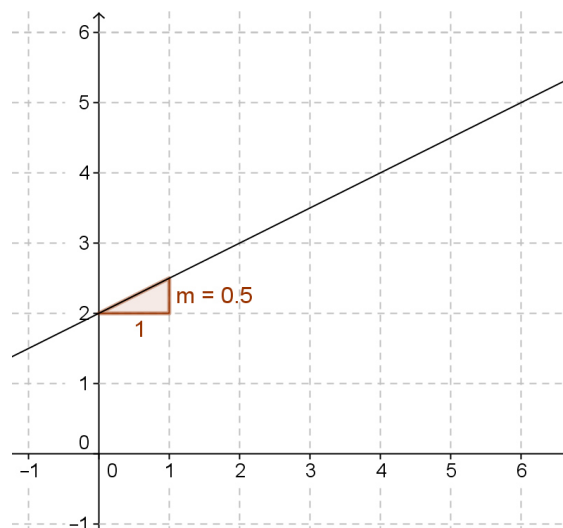
This is the kind of relation between two proportional variables when the initial value is different from zero. For example, the price of a taxi trip is proportional to the distance travelled but at the same time, it has an initial quantity added.

For example, the price of a taxi is €2 –fixed quantity- plus €0.5 per kilometre.

The formula is a first-degree function, that is, $y = mx + b$. Now 'm' is the constant of proportionality and b is the initial value.

Write the formula for the example:

The graph is a straight line whose slope is 'm'; 0.5 in the example, and whose ordinate in the origin is 'b'; 2 in the example.



'm' value is called the slope or gradient and indicates how steep the line is. 'b' value is called the y-intercept, the point that the line cuts the y-axis.

FUNCTION FROM TWO POINTS

Two points or data are enough information to get the formula for an affine function.

Let's say $A(x_1, y_1)$ and $B(x_2, y_2)$ are two whatever points from the function (chart).

We get the slope thus: $m = \frac{y_2 - y_1}{x_2 - x_1}$.

After that, it is easy to work out 'b' from any point A or B by replacing the previous value into the general formula $y = mx + b$ and clearing 'b'.

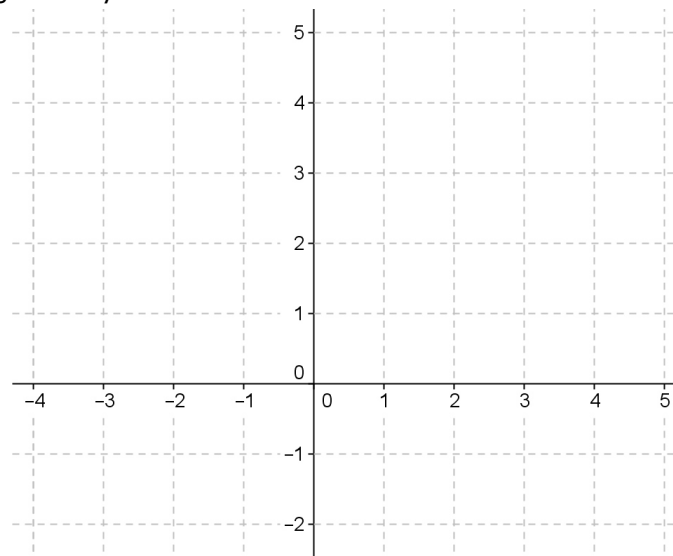
Work out the formula of an affine function that has (2, 7) and (3, 9) as values.

HORIZONTAL AND VERTICAL LINES

A horizontal line has a simple equation: $y = b$; that is, its slope is 0. It is a constant function.

A vertical line is simple also: $x = a$; that is, 'y' could be whatever value but 'x' always has the same value 'a'.

Draw the straight lines $y = 3$ and $x = -2$.

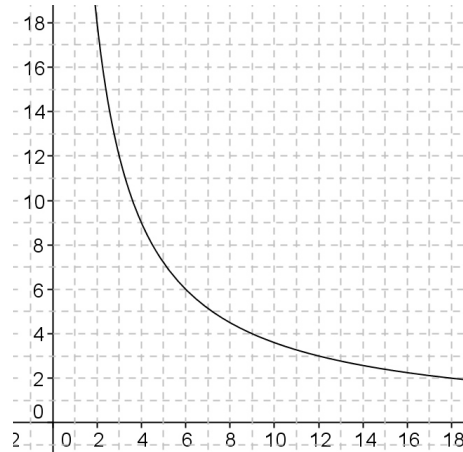


5. INVERSE PROPORTIONALITY FUNCTION. HYPERBOLAS

It is the relation between two magnitudes which are in inverse proportion. For instance; to rent a van for a trip is €36. The number of passengers and the price paid by each person are in inverse proportion.

GRAPH

The graph is a hyperbola.



Mark on the chart the corresponding price in the following situations: 2 passengers, 4 passengers, 6 passengers, 9 passengers. Circle the corresponding value on y-axis- for each.

EQUATION OR FORMULA

Since $x \cdot y = k$ in the inverse proportionality, the formula is $y = \frac{k}{x}$. We only have to clear 'y' in the first equality.

FUNCTION FROM ONE POINT OR DATA

One point or data is enough information to get the formula for an inverse proportion.

Being that $A(x, y)$ is whatever point from the function (chart) we get the constant thus:

$$k = x \cdot y$$

For example, 8 friends gave 6 euros each to buy a present to another. What is the formula between the number of friends and the contribution for the same present?

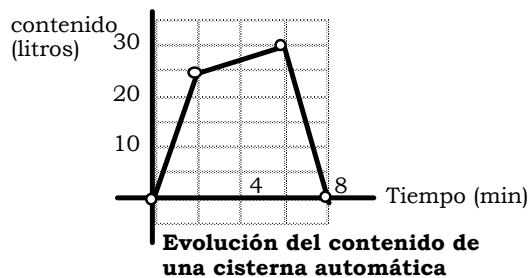
PROBLEMS AND EXERCISES

1. FUNCTION

1. Draw in a system of coordinate axes the following points:
 $A(-1, 0)$; $B(0, 1)$; $C(1, 0)$; $D(3, 0)$;
 $E(2, -1)$; $F(1, -1)$; $G(1, -2)$; $H(0, -1)$; $I(0, 0)$
 Join them in order to create a figure.

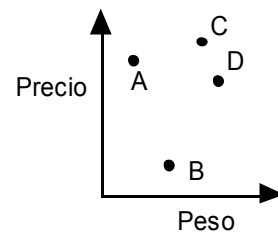


2. In this graph:
 a) Write the variables and its units.
 b) Make a table with the points that you see in the graph.
 c) Which is the maximum value?
 d) What are the increasing interval and the decreasing interval?
 e) Write the process that describes this graph.

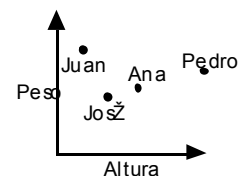


3. Write what is the independent variable for these functions. In other words: what does each written variable depend on?
 a) The time it takes to fill a 100 litre water drum in a water source.
 b) The length or circumference of a wheel.
 c) The tax I pay buying a car.
 d) The time for an object to reach the ground when we drop it.
 e) The width of a book.

4. We can see the price and the weight of several products of a supermarket in the next graph. Identify each of them by the data you have to the right.
- The most expensive item is the sausage.
 - The heaviest items are the beans.
 - The cheapest item is the pasta.
 - The lightest item is the tuna.



5. In this graph you can see the relationship between the height and the weight of some people.
 Who is the tallest?
 Who is the shortest?
 Who is the heaviest?
 Who is the lightest?
 Order people you can see in the chart according to their height.



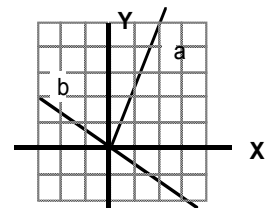
- 6.**
- What are the variables for the following functions?
 - What are the independent and dependent variables?
 - What is the formula for each one?
 - A hair dryer heats the air 10°C . x –input temperature-; y –output temperature-
 -
 -
 -
 - The kilogram of apples is €0.90
 -
 -
 -
 - The train costs €0.10 per kilometer plus €2 of insurance.
 -
 -
 -
 - The surface area of a square is equal to the side squared.
 -
 -
 -

2. LINEAR FUNCTION. DIRECT PROPORTIONALITY

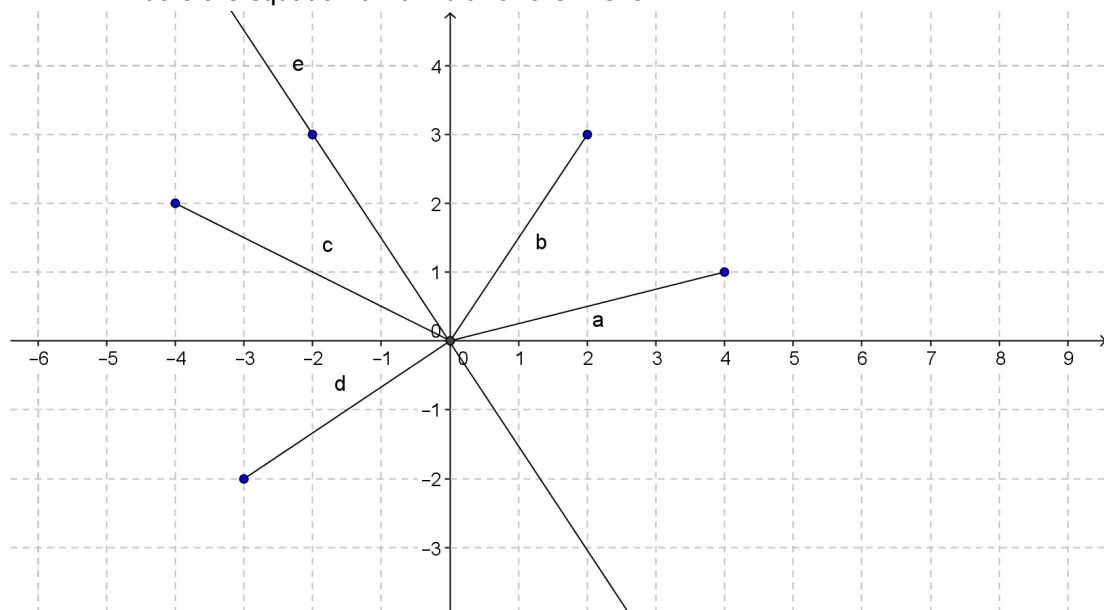
- 7.** a) Complete the next chart in order to get a direct proportionality:

x	2	3	5	7
y			12	

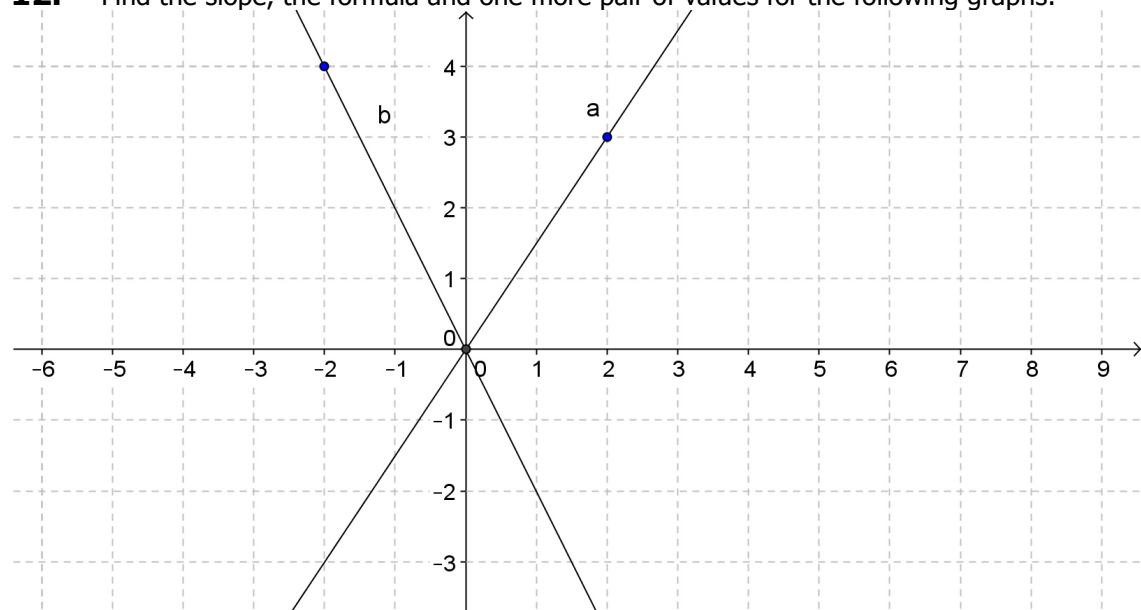
- What is the constant of proportionality?
 - What is the linear function formula?
- 8.** A person weight 60 kg and burns 2kcal per 5 minutes.
- Do a table that relates both variables. Write 5 values at least.
 - Draw the graph.
- 9.** Find the slope of the following lines
What is the formula for the functions they represent?
- 10.** Make a chart of values (three values) and the corresponding graph for the following functions:
- $y = 5/3x$
 - $y = -1'5x$



- 11.** Work out the slope for the following segments and straight line:
What is the equation of formula for the line 'e'?



- 12.** Find the slope, the formula and one more pair of values for the following graphs:



3. AFFINE FUNCTION

13. Using the graph answer the following questions:

- Make a table with the two values A and B.
- Calculate the slope.
- Work out the formula.

14. Draw the graph of the function: $y = -3x + 5$

15. The formula of a function is $y = -2x + 3$.

- How much is its slope and its y-intercept?
- Do a table with 3 values.
- Draw the graph.

16. Calculate the formula of a first degree function which takes the following values:

x	y
2	4
5	10

17. Draw the line through points A (2, 1) and B (5, 7). Find the formula.

18. Using the graph answer the following questions:

- Do a table with the two values A and B.
- Calculate the slope.
- Work out the formula.

19. Calculate the function formula of degree 1 which takes the following values:

x	y
2	5
4	8

20. Draw the line through points: A(-3, 0) and B(4, -2). Find the formula.

21. Draw the straight line through points: A(-3, 0) and B(4, -2). Find the formula.

22. The formula of a function is $y = 2x - 6$.

- How much is its gradient and its y-intercept?
- Do a table with 3 values.
- Draw the graph.

23. Draw the straight line through points: A(-2, 3) and B(0, -2).

24. Find the formula for the linear function that passes through the points: A(-2, -3) y B(6, 1). Draw the graph.

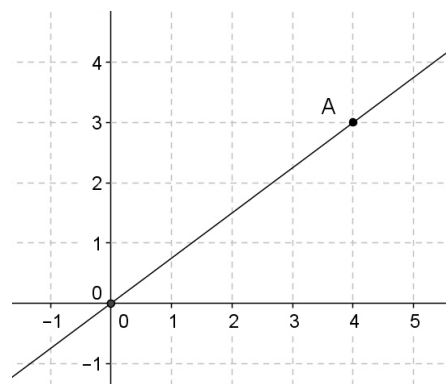
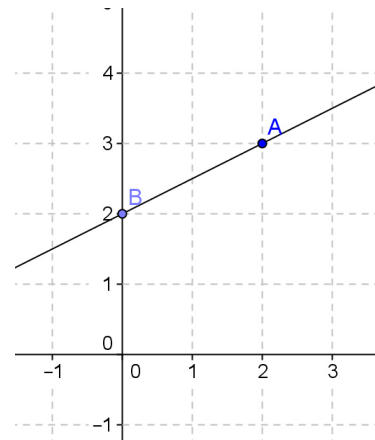
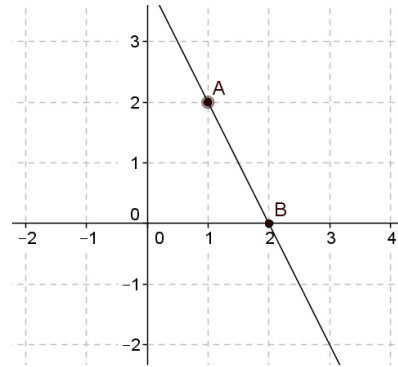
25. Using the graph answer the following questions:

- Do a table with two values.
- Calculate the slope.
- Work out the formula.

26. Draw the straight line through points: A(3, -4) and B(0, 2). Find the formula.

27. The formula of a function is $y = 4x - 3$.

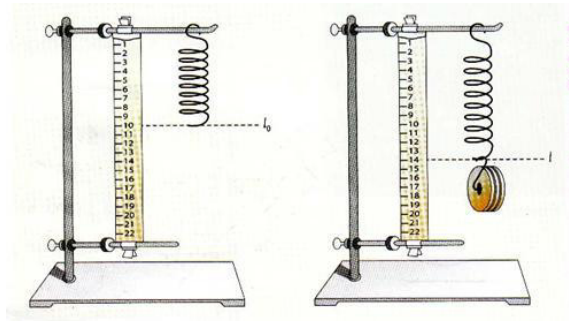
- How much is its slope and its y-intercept?
- Do a table with 3 values.
- Draw the graph.



- 28.** The formula of a function is $y = -3 + 5x$.
- How much is its gradient and its y-intercept?
 - Do a graph with 3 values.
 - Draw the graph.

- 29.** The formula of a function is $y = -2x + 3$.
- How much is its slope and its y-intercept?
 - Do a table with 3 values.
 - Draw the graph.

- 30.** A 10 inch spring hangs from the ceiling. Its elongation is 4 inches per hanging kg. We study the function that establishes the relation between the total length of the spring according to the weight set.
- Make a table with 3 values.
 - Find the function formula
 - Represent it.



- 31.** The price for a taxi is €1.5 plus €2 per kilometer. If we call x the number of kilometres travelled and y the tour price. What is the formula for the function?



- 32.** The price for a taxi is €2 and €3 per kilometer. If we call x the number of kilometres travelled and y the tour price.
- Do a table with three values.
 - Draw a graph.
 - What is the formula for the function?
- 33.** A train ticket costs €2 as insurance plus 6 cents per km.
- Write a table that relates the distance travelled with the price. Use 100 km as the unit of distance.
 - Draw a graph
 - Write the formula.
 - How much is a 1800 km trip?
- 34.** A repair costs €5 for travel expenses plus 12 € per hour of work. Find the formula for the function that relates the cost with the time spent on the repair.
- 35.** I paid €10 to repair my car for the flat fee plus €18 for every worked hour.
- Make a table that relates the working hours with the price.
 - Make a graph.
 - Find the function formula.
 - How much will a repair be if it takes six hours and a half?
- 36.** I paid 3 euros for a 100 Km trip and 7 euros for a 200 Km trip. The price is the result of adding the insurance plus a quantity per kilometer.
- Make a table with the two above values.
 - What is the price per km?
 - What is the insurance fee?
 - What is the formula that relates the price with the travelling distance?

4. ***INVERSE PROPORTIONALITY FUNCTION. HYPERBOLAS***

37. Represent the function: $y = \frac{-4}{x}$

38. The formula of a function is: $y = \frac{-6}{x}$

- Make a table. Take intelligent values to get integer results.
- Draw a graph.

39. The formula of a function is: $y = \frac{-8}{x}$

- Make a table. Take intelligent values to get integer results.
- Draw a graph.

40. The formula of a function is: $y = \frac{-12}{x}$

- Make a table. Take intelligent values to get integer results.
- Draw a graph.

41. A group of friends want to hire a pub to celebrate a party. The fee is €100. Make a table that shows the relation between the number of participants and each contribution. Draw a graph. What is the function formula?

42. There is a group of friends buying a football 20€.

- Make a table that relates the number of friends with each contribution. Take intelligent values to get integer results.
- Draw the corresponding graph.

43. A group of friends want to hire a pub to celebrate a party. The fee is €64.

- Make a table that shows the relation between the number of participants and each one contribution.
- Draw a graph.
- What is the function formula?

